# Universal Gravitation and Kepler's Laws

Objectives:

• Discuss Newton's universal law of gravity, and understand that it is an attractive force between two particles separated by a distance, r.

• State and interpret each of Kepler's three laws of planetary motion.

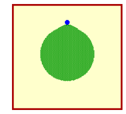
• Use a sketch to illustrate the motion of a typical planet and calculate orbital periods using tabulated values of planetary data.

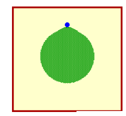
• Describe the nature of Newton's Universal law of gravity and the method of deriving Kepler's third law from this law of circular orbits.

# The Epiphany of The Apple

*In addition to developing his three laws of motion, Newton also investigated the motion of the planets and the Moon.*

- Newton's Law of Gravity is an extension of projectile motion.
  - Let's say we shoot a cannon ball from the top of a mountain. What will happen?
    - Falls to the ground!
  - What happens if it is shot faster ?
    - Eventually makes it all the way around!

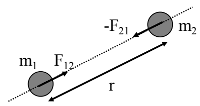




*The force keeping the moon in orbit is the same force that pulls an apple toward Earth!*

# Newton's Universal Law of Gravity

$$F = G\frac{m_1 m_2}{r^2}$$




Where…

$m_1$ , $m_2$ - masses of the two objects (kg)

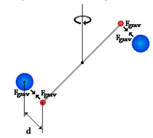
r - distance between the two centers of masses (m)

G - Universal Gravitational constant (6.67 x $10^{-11}$ $Nm^2/kg^2$)

- Since we have two objects interacting, what force is this?
  - Based on Newton's 3rd Law, all forces come in pairs.
  - Law of gravity gives the magnitude of the attractive force between the objects.
    - Directed toward each other along the line connecting them!

# The Cavendish Experiment

- Newton figured out that the force between the moon and the Earth was diluted by distance and that the relationship between force and distance was an inverse square.
  - Also figured out that the force is proportional to the product of the masses.
  - DID NOT figure out the gravitational constant.



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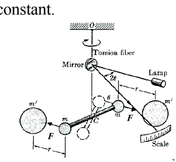



Current

Cavendish was trying to figure out the structure of the Earth, so he needed its mass. He was able to do this if Newton's law of gravity proved correct.

*He calculated the value of G to be 6.71 x $10^{-11}$ $Nm^2/kg^2$ which compared to the accepted value of 6.672 x $10^{-11}$ $Nm^2/kg^2$ was only off by about 1%.*

# Weighing The Earth

*Cavendish's experiment was referred to as weighing the Earth because the results allowed him to figure out the Earth's mass.*

$$F_g = G\frac{mM_E}{r^2} \quad \text{and} \quad F_g = mg$$

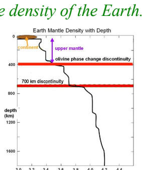
$$\cancel{m}g = G\frac{\cancel{m}M_E}{r^2} \longrightarrow \boxed{g = G\frac{M_E}{r^2}}$$

$$M_E = \frac{gr^2}{G} = 5.98 \times 10^{24}\ \text{kg}$$

*Using this value, Cavendish was able to calculate the average density of the Earth.*

$$\rho_E = \frac{M_E}{\frac{4}{3}\pi r^3} \longrightarrow \rho_E = 5500\ \text{kg/m}^3$$

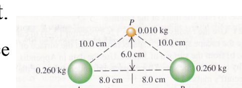
$$\rho_{crust} = 2700\ \text{kg/m}^3$$




*To get this average, the core's density must be greater!*

# Using The Universal Law

- The Universal Law of Gravity describes the interaction between two ***particles***.
  - Most of the objects that we deal with are a collection of particles, not individual particles.
- Law works when the objects are spherical.
  - Acts the same as if everything was condensed to the center.
    - Works well for most moons, planets, and stars.
- Since Gravity is an attractive force between two objects of mass, there can be more than one force of gravity acting on a given object.
  - Use the law to determine the force of gravity for each interaction.
    - Add the forces using vector addition.



# What's The Attraction?

- Let's say it's Valentines Day, and a couple is standing on the dance floor. If the mass of one is 60 kg while the other is 75 kg, what is the force of gravity between them if they are 30 cm apart?

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = (6.67 \times 10^{-11}\ Nm^2/kg^2)\frac{(60\ kg)(75\ kg)}{(.30\ m)^2}$$

$$F_g = 3.3 \times 10^{-6}\ N$$

- If the dance floor is frictionless, how long would it take for them to reach each other?

$a_1 = \frac{F_g}{m_1}$ $a_2 = \frac{F_g}{m_2}$

$x = 1/2at^2 + v_o t + x_o$

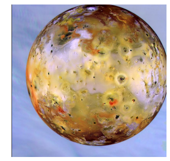
$x_P = 1/2a_1t^2 + 1/2a_2t^2$

$0.60 = (5.5\times10^{-8})t^2 + (4.4\times10^{-8})t^2$

$t = 2462\ s = 41\ min$

# What's the Acceleration?

- Jupiter's moon, Io, has a mass of $8.93 \times 10^{22}$ kg and has a diameter or 3630 km. What is the acceleration due to gravity experienced by an object at Io's surface?



Info:

$m = 8.93 \times 10^{22}$ kg

$d = 3630$ km $\longrightarrow$ $r = 1815$ km

$\longrightarrow$ $r = 1{,}815{,}000$ m

$F_g = G\frac{mM_I}{r^2}$ and $F_g = mg$

$$g = G\frac{M_E}{r^2} \longrightarrow g = (6.67 \times 10^{-11}\ Nm^2/kg^2)\frac{8.93 \times 10^{22}\ kg}{(1{,}815{,}000\ m)^2}$$

$g = 1.81\ m/s^2$

# Out In Space

- The word "planet" means wandering star because the planets seem to wander across the night sky in an unpredictable way.
  - The most confusing motion was that of Mars.
    - Sometimes it would move backward across the sky and then return on its course.
      - Known as retrograde motion.

- Tycho Brahe spent his entire life collecting data on the position of the planets.
  - With his death, this information was passed to his apprentice, Johannes Kepler, who used it to develop 3 empirical laws that describe the motion of the planets.

# Kepler's Laws

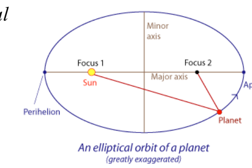
All three empirical laws were developed looking at the planets that orbited the sun. They have been found to be more universal in that they can be applied to satellites orbiting the planets.

1. Law of Orbits 2. Law of Equal Area 3. Law of Periods

# Law 1: Law of Orbits

***States:*** *All planets move in elliptical orbits, with the sun at one focus.*

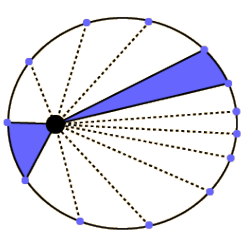
Supported by: Energy, Angular Momentum, and law of gravity.




# Law 2: Law of Equal Area

***States:*** *A line that connects a planet to the sun sweeps out equal area in equal time.*

Supported by: Conservation of Angular Momentum



Planets move faster at perihelion when closer to the sun.

Planets move slower at aphelion when farther from the sun.

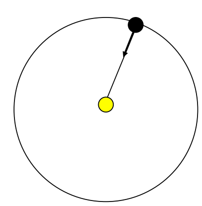
# Law 3: Law of Periods

***States:*** For circular orbits, *the square of the period of a planet is proportional to the cube of the semimajor axis of its orbit.*

$T^2 = Ka^3$ Where K is a constant.

Supported by: Law of Gravity

*In a circular orbit, the force of gravity acts as the centripetal force!*



$F_g = G\frac{m_E m_S}{R^2}$ and $F_g = m_E a_c$

$$m_E a_c = G\frac{m_E m_S}{R^2} \longrightarrow \frac{v^2}{R} = G\frac{m_s}{R^2}$$

# What About the Period?

***Recall:*** The period of rotation or revolution is the time it takes to complete one complete cycle or revolution.

$$v = \frac{2\pi R}{T}$$

Substituting in…

$$\frac{v^2}{R} = G\frac{m_s}{R^2} \longrightarrow \left(\frac{2\pi R}{T}\right)^2 \frac{1}{R} = G\frac{m_s}{R^2}$$

$$\frac{4\pi^2 R^2}{T^2}\frac{1}{R} = G\frac{m_s}{R^2} \longrightarrow \frac{4\pi^2 R}{T^2} = \frac{Gm_s}{R^2} \longrightarrow 4\pi^2 R^3 = Gm_s T^2$$

$$\boxed{T^2 = \frac{4\pi^2}{Gm_s}R^3} \qquad K = \frac{4\pi^2}{Gm_s}$$

The period of a planet or satellite is independent of its mass!

# Homework!

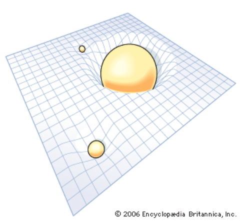
*Section 4.7, 5.5-5.6*

- Practice:
  - Ch 4: 25, 27, 31
  - Ch 5: 33, 35, 37
- Homework!
  - Ch 4: 20, 23, 28, 29
  - Ch 5: 31, 36, 38, 39

# Newton's Not Perfect!

- There are some flaws in Newton's concept of gravity that led Einstein to develop a different idea.
  - According to Einstein…

*Gravity is not a force but rather the result of a curvature of space-time by mass.*




The amount the space-time continuum is distorted is dependent on the mass.

Often represented as a cloth with large objects resting in it.

# The Graviton

- Physicists now believe that gravity is the result of mass-less particles known as gravitons.
  - Existence has been predicted by string theory.

Which is correct?

Space-time curvature or the graviton?

**We Don't Know Yet!**